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Urdhva-Tiryagbhyam Untapped Potential in Division Operations: An Examination

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Abstract: A distinctive and effective method for resolving mathematical issues is provided by Vedic Mathematics, which is based on the ancient Indian texts. Among its many strategies, it offers incredibly quick ways to perform fundamental operations like division, multiplication, subtraction, and addition. These sutra- or aphorism-based techniques reduce errors and simplify difficult computations. In this essay, we concentrate on the division operation, which is frequently regarded as one of the trickier and more time-consuming operations in algebra and arithmetic. By means of the sutra Urdhva-Tiryagbhyam (which means "vertically and crosswise"), Vedic Mathematics provides a sophisticated substitute for the laborious and error-prone division techniques found in conventional mathematics. This sutra has historically been used for multiplication, but it can also be cleverly modified to divide with amazing ease and speed. We illustrate the practical application of Urdhva-Tiryagbhyam to division issues using a number of well-chosen examples from algebra and arithmetic. To give a comprehensive grasp of the procedure and to demonstrate the effectiveness of the sutra, each example is described in detail. This method is very helpful in academic, competitive, and real-world settings since it not only saves computing time but also improves accuracy. Further research into the uses of Vedic mathematics in contemporary education and problem-solving is made possible by the studies described in this paper. The methods covered have a lot of room for further study and can be incorporated into more comprehensive frameworks for teaching mathematics to improve students' computational abilities.

keywords: Division, Urdhva-Tiryagbhyam sutra, Vedic Math, Arithmetic, Algebra

Introduction

Vedic Mathematics is a system of mathematical techniques based on ancient Indian scriptures called The Vedas, rediscovered between 1911 and 1918 by Shankaracharya Jagadguru Shri Bharati Krishna Tirthji published in 1965.

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Jagadguru Shankaracharya Swami Bharati Krishna Tirthji was born in Tamil brahmin family in Tirunelveli, Madras Presidency on 14 March,1884 and took samadhi on 2 February, 1960. His early childhood name was Venkataraman Shastri. First, he became Shankaracharya of Dwarka Math, then became 143rd Shankaracharya and supreme monk of Govardhan Math in Puri in Odisha from 1925 through 1960. He worked and wrote extensively on different areas.

Vedic Mathematics aims to simplify calculations, improve speed and foster a deeper understanding of mathematical concepts. It is applied to arithmetic, algebra, geometry, calculus etc.

Urdhva-Tirygbhyam (means vertically and cross wise) is one such sutra used for multiplication and division. In this paper this sutra is applied on Division- arithmetic as well as algebra.

Representation of the sutra –

For Division:

DIVIDEND = DIVISIOR *QUOTIENT + REMAINDER

The same pattern has been followed in this research work.

<u>Taking Arithmetic Division first</u>

Case 1: 2D*2D

Example 1: 1289 ÷ 43

1289 dividend

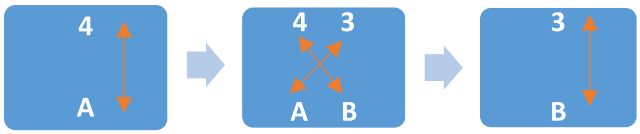
43 divisors

Dividend (D^d) = Divisor *Quotient + Remainder

1289 = 43*? + R

As per the sutra-

```
  4 3
* A B
1 2 8 9
```



4 3

*A B

1 2	8	9
4A	4B + 3A	3B
1 2	4 8	6 9
-8	- 6	- 2 7
	4 2	
	- 3 6	

A= 2

4A = 4*2= 8

4B + 3A = 48

4B = 48 – 6 = 42

B = 42/4 = 9, 6 B = 9, 6(remainder)

3B = 69

3*9= 69

27 = 69

69-27 = 42 (final remainder)

Methodology/ Process:

Step 1: first vertically multiply from left hand side ten's place with ten's place i.e. 4*A

Now compare 12 with 4A

Let's assume A = 3	A = 2
4*3 = 12	4*2 = 12
12 = 12	8 = 12
12 – 12 = 0	12 – 8 = 4 (remainder adjust to next step, therefore 8 will become 48)

Step 2: now cross multiply ten's place with one's place and one's place with ten's place i.e. 4B & 3A

When A = 3	when A = 2
Compare 8 with 4B + 3B	compare 48 with 4B + 3B
4B + 3A = 8 (remainder from step 1 is 0,	4B + 3A = 48
So compare only 8)	4B + 6 = 48
Put A =3	4B = 48 – 6 = 42

4B + 9 = 8

B = -1/4 (as value of B couldn't be negative,
So, in that case drop value of A in step 1)

B = 42/4 = 9, 6

B = 9 (as maximum we can take single digit quotient)
6 (remainder to be adjusted in next step, therefore 9 will become69)

Step 3: vertically multiply one's place with one's place i.e. 3*B

Compare 69 with 3B

3B = 69

Put value of B = 9

3*9 = 69

27 = 69 (always remember left hand side should always be smaller than right hand side, in arithmetic)

69 – 27 = 42 (final remainder)

Now put all the values in defined pattern of division: A = 2, B = 9

Dividend (D^d) = Divisor * Quotient + Remainder

1289 = 43 * 29 + 42 (answer)

Here you observed in case 1 example, dividend is of 4 digits.

As per the sutra 4 digits product bring two possibilities that is 2digit*2digit or 3digit*2digit

In case 1 we have discussed 2digit*2digit

Now in case 2, 2^{nd} possibility 3digit*2digit will be discussed.

Case 2: **2digit*3digit**

Example 2: 8760 ÷ 43

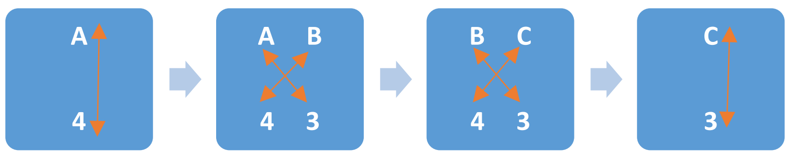
Dividend (D^d) = Divisor * Quotient + Remainder

8760 = 43 *? + R

As per the sutra

```
  A B C
 *4 3
______
```

8 7 6 0



A B C

* 4 3

8	7	6	0
4A	3A+4B	3B+4C	3C
8	$_0$7	$_1$6	$_4$0
-8	- 6	- 0	- 9
	1	16	31 (final remainder)
	-0	- 12	

A= 2

4A = 8

8 = 8, 8 - 8 = 0

3A + 4B = 7

4B = 7-6

B = 1/4, B= 0, 1(remainder)

3B + 4C = 16

4C = 16

C = 16/4, C = 3, 4(remainder)

3C = 40

9 = 40

40-9=31(final remainder)

Methodology/ Process:

Step 1: starting from left hand side vertically multiply 4*A

Compare 8 with 4A

4A = 8

Assume value of A = 2

4*2 =8

8 = 8 , 8 – 8 = 0 (remainder zero, so nothing will be carry forwarded)

Step 2: now cross multiply 3A+4B and compare with 7

3A + 4B = 7, put value of A

4B = 7 – 6 = 1

B = 1/4, B = 0, 1

Here we've got value of B i.e. 0(zero) as numerator is smaller than denominator, so 1 will be kept as remainder and adjusted to next step, therefore 6 will become 16

Step 3: again, cross multiply ten's place with one's place and one's place with ten's place i.e. 3B + 4C

Now compare 16 with 3B + 4C, put value of B

4C = 16 – 0	4C = 16 - 0
C = 16/4	C = 16/4
if C = 4, 0 (remainder)	if C = 3, 4(remainder adjusted to next step & 0 become 40)

Step 4: now vertically multiply one's place to one's place i.e. 3*C, and compare

If value of C = 4	if value of C = 3
3C = 0	3C = 40
12 = 0	9 = 40
0 – 12 = - 12 (as per the mathematics rule,	40 – 9 = 31 (final remainder)

in arithmetic division remainder couldn't be negative)

Put all the values in division pattern: A = 2, B = 0, C = 3

Dividend = Divisor * Quotient + Remainder

8760 = 43 * 203 + 31 (answer)

Points to Remember:

- How to identify whether the 4 digits dividend is the result of 2digit*2digit or2digit*3digit?
- So just focus here- sutra applied here is Vilokanam (means mere observation)
- In case 1 (1289 ÷ 43), compare 12 & 4 quotient you got was single digit and remainder was also smaller.

Whereas

- In case 2 (8760 ÷ 43), compare 87 & 4 quotient can be taken only upto 9

4*9 = 36, 87 – 36 = 51 remainder is too high that can create problem in further calculation and can mislead you in getting answer or if clearly said, you might not get answer. So this perfectly indicates the case of 3digit*2digit.

ALGEBRAIC DIVISION EXAMPLES

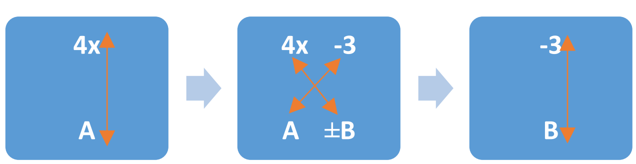
Example 3: $8x^2 - 2x + 9 \div 4x - 3$

As per the sutra

4x – 3

A ± B

$8x^2 - 2x + 9$



4x -3

*A ±B

$4xA = 8x^2$

$A = 8x^2/4x$

A = 2x

4xB - 3A = -2x

4xB – 6x = -2x

4xB = -2x + 6x

B = 4x/4x,

B=1

-3B = 9

-3=9

9+3 = 12 (final remainder)

$8x^2$	$- 2x$	$+ 9$
$4xA$	$4xB - 3A$	$-3B$

Methodology/ Process:

Step 1: starting from the left-hand side, vertically multiply 4x * A

Compare $4xA = 8x^2$ from here we've got value of A

$A = 8x^2/4x$

$A = 2x$

Step 2: cross multiply ten's place with one's place and one's place with ten's place i.e. 4xB & -3A

Compare -2x with 4xB + (-3A) and put value of A

$4xB - 3A = -2x$

$4xB = -2x + 6x$

$B = 1$

Step 3: finally, vertically multiply one's place to one's place in the right-hand side i.e. -3*B

Compare 9 with -3B and put the value of B

$-3B = 9$

$-3 = 9$

$9 + 3 = 12$

Now in this way we've got the values of both the variables and got final remainder also. So put all the values in division pattern: A = 2x, B = 1

As you can see this is the case of 2digit*2digit

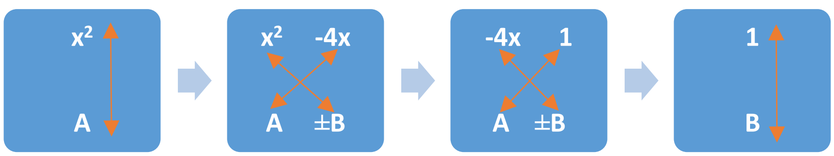
Dividend = Divisor * Quotient + Remainder

$8x^2 - 2x + 9 = (4x-3) * (2x + 1) + 12$ (answer)

Example 4: $x^3 + 4x^2 - 3x + 8 \div x^2 - 4x + 1$

As per the sutra

$$\begin{array}{c} x^2 - 4x + 1 \\ {}^*A \pm B \\ \hline x^3 + 4x^2 - 3x + 8 \\ \hline \end{array}$$



$$x^2 - 4x + 1$$

$${}^*A \pm B$$

x^3	$+ 4x^2$	$- 3x$	$+ 8$
Ax^2	$bx^2 - 4xA$	$A - 4xB$	B

$Ax^2 = x^3$

$A = x^3/x^2$

$A = x$

$Bx^2 - 4xA = 4x^2$

$Bx^2 - 4x^2 = 4x^2$

$Bx^2 = 8x^2$

$B = 8$

$A - 4xB = -3x$

$x - 32x = -3x$

$-3x + 31x = 28x$

$B = 8$

$8 = 8$

8-8=0 (no remainder left)

Methodology/ Process:

Step 1: starting from left-hand side vertically multiply $A*x^2$

Compare x^3 with Ax^2, then the value of A=x

Step 2: cross multiply $B*x^2$ & A*(-4x)

Compare $4x^2$ with $Bx^2 – 4xA$, put the value of A

$Bx^2 – 4x^2 = 4x^2$

$Bx^2 = 8x^2$

B = 8, from here we have got the value of B

So here we have got the values of both the variables and answer of quotient part is (x+8).

Now further steps are for getting remainder, if any.

Step 3: again, cross multiply A & -4x*B

Compare -3x with A – 4xB, put the values of A & B

x -32x = -3x

-3x + 31x = 28x

Step 4: finally, from right-hand side vertically multiply B*8, put the value of B

B = 8

8 = 8

8 – 8 = 0(no remainder left)

Put all the values in division pattern: A = x, B = 8

Dividend = Divisor * Quotient + Remainder

$x^3 + 4x^2 – 3x + 8 = (x^2 – 4x + 1) * (x + 8) + 0$

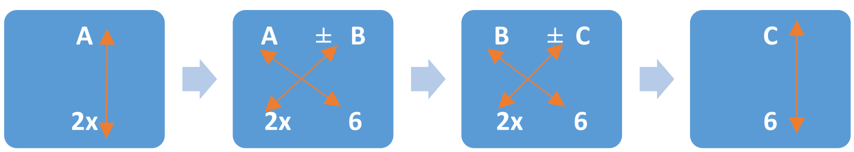
or

$x^3 + 4x^2 - 3x + 8 = (x^2 - 4x + 1) * (x + 8)$ (answer)

Example 5: $\boldsymbol{2x^3 + 5x^2 + 8x + 7 \div 2x + 6}$

As per the sutra

$$\begin{array}{lll} A & \pm B & \pm C \\ *2x & +6 & \\ \hline \multicolumn{3}{c}{2x^3 + 5x^2 + 8x + 7} \\ \hline \end{array}$$



A ± B ± C			
*2x 6			
$2x^3$	$+ 5x^2$	$+ 8x$	$+7$
A2x	6A ±2xB	6B ± 2xC	6C

$A \times 2x = 2x^3$

$A = 2x^3/2x$

$A = x^2$

$6A + 2xB = 5x^2$

$2xB = 5x^2 - 6x^2$

$B = -x^2/2x$

$B = -x/2$

$6B + 2xC = 8x$

$-6x/2 + 2xC = 8x$

$C = (8x + 3x)/2x$

$C = 11/2$

$6C = 7$

$66/2 = 7$

$7 - 33 = -26$ (final remainder)

Methodology/ Process:

Step 1: starting from left-hand side vertically multiply A*2x

Compare $2x^3$ with A2x and get the value of A

$A2x = 2x^3$

$A = 2x^3/2x$, $A=x^2$

Step 2: now cross multiply A*6 & 2x*B

Compare $5x^2$ with A6 & 2xB, put the value of A

$6x^2 + 2B = 5x^2$

$B = -x/2$, here we've got the value of B

Step 3: again, cross multiply 6*B & 2x*C

Compare 8x with 6B & 2xC and put the value of B

$-6x/2 + 2xC = 8x$

$C = 11/2$, here we have got the value of C, put this in the last step and get final remainder

Step 4: finally, from right-hand side vertically multiply 6*C

$6C = 7$

$66/2 = 7$

$7 – 33 = -26$ (final remainder), as we know in algebra values or remainders can be negative.

Put all the values in division pattern: $A = x^2$, $B = -x/2$, $C = 11/2$

Dividend = Divisor* Quotient + Remainder

$2x^3 + 5x^2 +8x +7 = (2x + 6) * (x^2 – x/2 + 11/2) – 26$ (answer)

Points to Remember:

- While doing algebraic division using the sutra urdhvatiryagbhyam how to analyze the answer part?
- So just focus here, i) check the highest power of x

ii) and the lowest power of x

- Now subtract the lowest power from the highest power and that is your answer.
- Like, in example 1: **$8x^2 - 2x + 9 \div 4x - 3$,** highest power of x is 2 and lowest power is 1, so 2-1 =1 means you need only power 1 i.e.(2x+1), x^1 as the answer.
- In example 2: **$x^3 + 4x^2 - 3x + 8 \div x^2 - 4x + 1$,** highest power of x is 3 and lowest power is 2, so 2-1 = 1, means x^1, (x+8) is the answer.
- In example 3: **$2x^3 + 5x^2 + 8x + 7 \div 2x + 6$,** highest power of x is 3 and lowest power is 1. So, 3-1 = 2 means here required power 2 as an answer (x^2 -x/2 +11/2).
- x(x^1) means power is 1.
- x^2 means power is 2.
- x^3 means power is 3….and so on….

References: The Urdhva-Tiryagbhyam sutra's application to division illustrates the Vedic mathematics' ease of use, accuracy, and speed. In addition to improving problem-solving abilities in academic and competitive settings, this method offers chances to incorporate traditional techniques into contemporary teaching, which promotes effectiveness and a deeper comprehension of mathematics**.**

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